Soft gluon emission process in the color-octet model for heavy quarkonium production

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The color-octet model has been used successfully to analyze many problems in heavy quarkonium production. We examine some of the conceptual and practical problems of the soft gluon emission process in the color-octet model. We use a potential model to describe the initial and final states in the soft gluon emission process, as the emission occurs at a late stage, after the production of the heavy quark pair. It is found in this model that the soft gluon $M1$ transition $^1S_0^{(8)} \rightarrow {}^3S_1^{(1)}$ dominates over the $E1$ transition $^3P_J^{(8)} \rightarrow {}^3S_1^{(1)}$ for J/ψ and ψ' production. Such a dominance may help resolve the questions of isotropic polarization and color-octet matrix element universality in the color-octet model. [S0556-2821(99)03721-2]

I. INTRODUCTION

Bodwin, Braaten, and Lepage [1] have recently developed the color-octet model based on nonrelativistic quantum chromodynamics for very massive quarks which allows a systematic calculation of inclusive heavy quarkonium production cross sections. It is assumed that in addition to the production of bound states by the color-singlet mechanism [2], bound states are produced by the color-octet mechanism whereby a $Q\bar{Q}$ pair in a color-octet state is first formed either by gluon fragmentation or by direct parton reactions, and the color of the pair is neutralized by emitting a soft gluon of low energy and momentum. The cross section for bound state ψ production is represented by

$$\sigma(\psi) = \sum_{ab} \sum_{c,J,L,S} F(ab \rightarrow {}^{2S+1}L_J^c) \frac{\langle 0|O(^{2S+1}L_J^c \rightarrow \psi)|0\rangle}{M_Q^d}, \quad (1)$$

where $ab = gg, q\bar{q}, \ldots$, are the parton combinations leading to production, and $c = 1,8$ are the color states of the ψ precursor. The quantity $F(ab \rightarrow {}^{2S+1}L_J^c)$ is the short-distance cross section for the production of a $Q\bar{Q}$ pair with quantum numbers $^{2S+1}L_J^c$ calculated from perturbative QCD (PQCD), based on the Feynman diagram for $ab \rightarrow (c\bar{c})_{cJLS} + X$. The matrix element $\langle 0|O(^{2S+1}L_J^c \rightarrow \psi)|0\rangle$ is the long-distance probability per unit volume for $^{2S+1}L_J^c$ to produce the final bound state ψ and d is the scaling dimension of the operator [1]. For example, for production from the color-singlet state $^{2L+1}L_J^{(1)}$ to the bound state ψ_{JLS}, which has the radial wave function $R_{JLS}(r)$, the matrix element is

$$\langle 0|O_1^{\psi_{JLS}}(^{2S+1}L_J^{(1)} \rightarrow \psi_{JLS})|0\rangle \propto \left| \left[\frac{d^L R_{JLS}(r)}{dr^L} \right]_{r \rightarrow 0} \right|^2. \quad (2)$$

For production from the color-octet state $^{2S'+1}L'_{J'}^{(8)}$ to the color-singlet bound state ψ_{JLS}, the color-octet matrix element $\langle \mathcal{O}_8^{\psi_{JLS}}(^{2S'+1}L'_{J'}) \rangle$ specifies the probability per unit volume for the $Q\bar{Q}$ color-octet state $^{2S'+1}L'_{J'}^{(8)}$ to emit a soft gluon in the transition to the bound color-singlet state ψ_{JLS}.

As the soft gluon emission takes place on a nonperturbative QCD time scale and involves nonperturbative QCD, the color-octet matrix elements have been treated as phenomenological parameters [3–11]. Matrix elements have been extracted to yield good agreement with the data of the Collider Detector at Fermilab (CDF) for high p_T heavy quarkonium production in $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV [5,13] and with the fixed-target data for pN collisions at energies up to $\sqrt{s} = 40$ GeV [6].

For the color-octet model to be a valid description, one should be able to describe the process of soft gluon emission in physical terms. We know the nature of the final state of observed quarkonium, but what is the nature of the initial color-octet state which emits a soft gluon? What is the nature of the soft gluon? What is the process of soft gluon emission? Which one of the gluon emission processes is more important in J/ψ and ψ' production? In addition to these conceptual questions, there are also the following practical questions which arise in the application of the color-octet model.

In the color-octet model for fixed target energies, the dominant production comes from the fusion of two gluons forming a $c\bar{c}$ color-octet pair in $^1S_0^{(8)}$ and $^3P_J^{(8)}$ states. The soft-gluon radiative transition $^1S_0^{(8)} \rightarrow {}^3S_1^{(1)}$ leads to an isotropic angular distribution of decay muons in the quarkonium rest frame; the $^3P_J^{(8)} \rightarrow {}^3S_1^{(1)}$ transition in contrast preferentially populates $J_z = \pm 1$ substates with a large transverse polarization, which leads to anisotropic angular distributions in the decay of the charmonium to muons. The muon angular distribution can be inferred from the magnitudes of the matrix elements $\langle \mathcal{O}_8^H(^1S_0) \rangle$ and $\langle \mathcal{O}_8^H(^3P_0) \rangle / m_c^2$. The J/ψ and ψ' production cross sections at fixed-target energies give the matrix elements [6]

$$\langle \mathcal{O}_8^{J/\psi}(^1S_0) \rangle + \frac{7}{m_c^2} \langle \mathcal{O}_8^{J/\psi}(^3P_0) \rangle = 3.0 \times 10^{-2} \text{ GeV}^3 \quad (3)$$

for J/ψ production and

$$\langle \mathcal{O}_8^{\psi'}(^1S_0) \rangle + \frac{7}{m_c^2} \langle \mathcal{O}_8^{\psi'}(^3P_0) \rangle = 0.5 \times 10^{-2} \text{ GeV}^3 \quad (4)$$

for ψ' production. On the other hand, the velocity counting rule of Ref. [1] gives an $\langle \mathcal{O}_8^H(^1S_0) \rangle$ of the same order as $\langle \mathcal{O}_8^H(^3P_0) \rangle / m_c^2$. If we set $\langle \mathcal{O}_8^H(^1S_0) \rangle = \langle \mathcal{O}_8^H(^3P_0) \rangle / m_c^2$, as suggested by the velocity counting rule of Ref. [1], the expected angular distribution of muons from J/ψ decay will not be isotropic [6]. The experimental data give an isotropic distribution for J/ψ and ψ' production in π-W collisions at 252 GeV [14,15] and 125 GeV [16].

For the color-octet model to be a valid description, the color-octet matrix elements $\langle \mathcal{O}_8^H(^{2S'+1}L'_{J'}) \rangle$ should be independent of the processes which produce $^{2S'+1}L'^{(8)}_{J'}$. However, there are unresolved questions concerning this universality of the color-octet matrix elements. High p_T CDF measurements at $\sqrt{s} = 1.8$ TeV give [5]

$$\langle \mathcal{O}_8^{J/\psi}(^1S_0) \rangle + \frac{3}{m_c^2} \langle \mathcal{O}_8^{J/\psi}(^3P_0) \rangle = 6.6 \times 10^{-2} \text{ GeV}^3 \quad (5)$$

for J/ψ production and

$$\langle \mathcal{O}_8^{\psi'}(^1S_0) \rangle + \frac{3}{m_c^2} \langle \mathcal{O}_8^{\psi'}(^3P_0) \rangle = 1.8 \times 10^{-2} \text{ GeV}^3 \quad (6)$$

for ψ' production. On the other hand, J/ψ and ψ' production cross sections at fixed-target energies give the matrix elements [6] listed in Eqs. (3) and (4). To check the universality of matrix elements, we need a relation between $\langle \mathcal{O}_8^H(^1S_0) \rangle$ and $\langle \mathcal{O}_8^H(^3P_0) \rangle$ for $H = \{J/\psi, \psi'\}$. If we again use the velocity counting rule of [1] to set $\langle \mathcal{O}_8^H(^1S_0) \rangle = \langle \mathcal{O}_8^H(^3P_0) \rangle / m_c^2$, the fixed-target matrix elements are a factor of 4(7) smaller than the CDF matrix elements for $J/\psi(\psi')$, as pointed out by Beneke *et al.* [6].

We would like to formulate the color-octet model in a form which will allow us to answer these conceptual and practical questions. We know that the final observed state can be described nonperturbatively in terms of a potential model. Since the emission of the soft gluon takes place at a long-time scale after the production of the $Q\bar{Q}$ pair, it is reasonable to describe the initial state which emits a soft gluon also in terms of a potential model. The soft-gluon emission matrix element can then be evaluated with wave functions of the initial and final states in this potential model and the density of color-octet states. From these formulations, we find that when a color-octet $Q\bar{Q}$ pair in the $^1S_0^{(8)}$ or $^3P_J^{(8)}$ state emits a very soft gluon to make a transition to the bound $H = {}^3S_1^{(1)}$ state, the probability for the $M1$ radiative transition $^1S_0^{(8)} \to {}^3S_1^{(1)}$ is much greater than that for the $E1$ transition, $^3P_J^{(8)} \to {}^3S_1^{(1)}$, and the velocity counting rule breaks down. We suggest that such a dominance of $\langle \mathcal{O}_8^{^3S_1}(^1S_0) \rangle$ over $\langle \mathcal{O}_8^{^3S_1}(^3P_J) \rangle / m_c^2$ for soft gluon radiation may help resolve the above questions of quarkonium polarization and matrix element universality.

This paper is organized as follows. In Sec. II, we formulate the model of quarkonium production in terms of the short-distance Feynman amplitude and wave functions determined from potential models. An explicit expression is obtained to relate the color-octet matrix element in the color-octet model to quantities which can be evaluated in the potential model. In Sec. III, we examine the potential between Q and $\bar{Q}$ interacting at large distances. When one allows for the effect of the spontaneous production of light quarks to break up the string joining Q and $\bar{Q}$, the linear potential is modified to become a screened potential. We use a screened potential to discuss bound states in Sec. III and continuum resonance states in Sec. IV. Section IV also outlines how we obtain continuum wave functions which are needed in the calculation of the cross section. Section V gives the multipole transition matrix elements. As an illustration, a simple $M1$ to $E1$ ratio is obtained for the production of J/ψ using approximate wave functions. Numerical results with more realistic potentials are obtained in Sec. VI to give the contributions from $M1$ and $E1$ transitions to J/ψ and ψ' production. It is found that the soft gluon $M1$ radiative transition $^1S_0^{(8)} \to {}^3S_1^{(1)}$ dominates over the $E1$ transition $^3P_J^{(8)} \to {}^3S_1^{(1)}$ for J/ψ and ψ' production. Section VII summarizes and concludes the present discussions.

II. HEAVY QUARKONIUM PRODUCTION FROM PARTON COLLISIONS

To obtain the cross section for the production of quarkonium bound states in a hadron-hadron collision, it suffices to focus on the production cross section in parton collisions, as the former can be obtained from the latter by folding the parton distribution functions of the colliding hadrons. We consider the collision of the parton a with the parton b to produce the initial $Q\bar{Q}$ pair. The heavy quarkonium production amplitude can be obtained by projecting out the state vector of the $Q\bar{Q}$ pair onto the quarkonium bound state. We shall show in detail how this is carried out to yield the quarkonium production cross section.

The production of a heavy quark pair is a fast process. We can follow the time evolution of the state vector of the $Q\bar{Q}$ pair. The state vector resulting from the collision of a and b at initial production time t_i is

$$|\Phi_{ab}(t_i)\rangle = |\Phi_{ab}^{Q\bar{Q}}(t_i)\rangle + |\Phi_{ab}^{Q\bar{Q}g}(t_i)\rangle + \cdots, \quad (7)$$

where

$$|\Phi_{ab}^{Q\bar{Q}}(t_i)\rangle = \int \frac{d^3Q}{(2\pi)^3 2E_Q} \frac{d^3\bar{Q}}{(2\pi)^3 2E_{\bar{Q}}} (2\pi)^4$$
$$\times \delta^{(4)}(P_i - P_f) \mathcal{M}(ab \to Q\bar{Q}) |Q\bar{Q}\rangle, \quad (8)$$

$$|\Phi_{ab}^{Q\bar{Q}g}(t_i)\rangle = \int \frac{d^3Q}{(2\pi)^3 2E_Q} \frac{d^3\bar{Q}}{(2\pi)^3 2E_{\bar{Q}}} \frac{d^3g}{(2\pi)^3 2E_g} (2\pi)^4$$
$$\times \delta^{(4)}(P_i - P_f) \mathcal{M}(ab \to Q\bar{Q}g) |Q\bar{Q}g\rangle. \quad (9)$$

Here, P_i and P_f are the initial and final four-momentum. The matrix elements $\mathcal{M}$ can be obtained in perturbative QCD using Feynman diagrams and renomalization procedures.

For a sufficiently large value of t, the initial state in Eq. (7) will evolve to become

$$|\Phi_{ab}(t)\rangle = \sum_x \int d\Phi_{\psi x}(2\pi)^4 \delta^{(4)}(P_i - P_f)$$

$$\times \mathcal{M}(ab \to \psi_{JLS}x)|\psi_{JLS}x\rangle, \qquad (10)$$

where x denotes the number of hard gluons, and $d\Phi_{\psi x}$ is the corresponding phase space volume element. For example, for $ab \to \psi$, $d\Phi_\psi$ is

$$d\Phi_\psi = \frac{d^3 P_\psi}{(2\pi)^3 2E_\psi}, \qquad (11)$$

and for $ab \to \psi g$ with the emission of a hard gluon, $d\Phi_{\psi g}$ is

$$d\Phi_{\psi g} = \frac{d^3 P_\psi}{(2\pi)^3 2E_\psi} \frac{d^3 g}{(2\pi)^3 2E_g}. \qquad (12)$$

The cross section for the production of the quarkonium state ψ_{JLS} by the collision of partons a and b is then

$$d\sigma(ab \to \psi_{JLS}x) = \frac{1}{4I}(2\pi)^4 \delta^{(4)}(P_i - P_f)$$

$$\times \sum_x |\mathcal{M}(ab \to \psi_{JLS}x)|^2 d\,\Phi_{\psi x} \qquad (13)$$

where $I = \sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}$. It is necessary to obtain $\mathcal{M}(ab \to \psi_{JLS}x)$ from $\mathcal{M}(ab \to Q\bar{Q}x)$ in order to calculate the production cross section. We shall show how this relation can be obtained in the present model description.

One can first separate out the center-of-mass motion and the relative motion of the $Q\bar{Q}$ pair in terms of their total momentum P and their relative momentum q. The heavy quark momentum is then $Q = P/2 + q$, the antiquark momentum is $\bar{Q} = P/2 - q$, and the state of the center-of-mass motion can be represented by the plane wave state $|Q\bar{Q}\rangle$ which is equivalent to $|qP\rangle$. They are related by the ratio of their normalization constants.

We look first at the situation when Q and $\bar{Q}$ are not interacting to introduce their interaction through a mutual potential V later on. Part of the phase space elements, $d^4Q\,d^4\bar{Q}$, in Eqs. (8) and (9) can be transformed into

$$\frac{d^4Q}{(2\pi)^3}\delta(Q^2 - m_Q^2)\frac{d^4\bar{Q}}{(2\pi)^3}\delta(\bar{Q}^2 - m_{\bar{Q}}^2)$$

$$= \frac{d^4P}{(2\pi)^3}\delta(P^2 - s)\frac{d^4q}{(2\pi)^3}\delta\left(\frac{P \cdot q}{2}\right), \qquad (14)$$

where $\sqrt{s}$ is the invariant mass of the $Q\bar{Q}$ system. To study the dynamics of the relative momentum, it is best to refer to the center-of-mass system where $P = (\sqrt{s}, 0)$, $q = (0, q)$, and the relative momentum q satisfies the following equation:

$$\epsilon_\omega^2 - q^2 - m_\omega^2 = 0, \qquad (15)$$

where

$$\epsilon_\omega = \frac{s - 2m_Q^2}{2m_P}, \qquad (16)$$

and m_ω is the reduced mass

$$m_\omega = m_Q^2/\sqrt{s}. \qquad (17)$$

Next, we consider the system with a mutual timelike vector interaction V. The equation of motion can be obtained from Eq. (15) by the canonical method of replacing ϵ_w with $\epsilon_w - V(r)$. The Klein-Gordon equation for the two-particle system under a mutual timelike vector interaction $V(r)$ is

$$\{[\epsilon_\omega - V(r)]^2 - q^2 - m_w^2\}\Psi(r) = 0. \qquad (18)$$

Similar two-body equations of constraint dynamics for more complicated cases with spin and very general types of interactions can be found in Ref. [17]. Because of the large mass of the heavy quark, it is convenient to use nonrelativistic $Q\bar{Q}$ wave functions and state vectors. The Klein-Gordon equation becomes the Schrödinger equation

$$\left\{\frac{q^2}{2m_\omega} + V(r) - \epsilon\right\}\Psi(r) = 0, \qquad (19)$$

where $\epsilon = \epsilon_\omega - m_\omega$ is the nonrelativistic measure of energy, and $m_\omega \sim M_Q/2$. The eigenvalue ϵ of the bound state is related to the masses by $\sqrt{s} = \epsilon + 2m_Q$. The bound state wave function can be written in the form

$$\Psi_{JLS}(r) = R_{JLS}(r)\mathcal{Y}_{JLS}(\hat{r}). \qquad (20)$$

The wave function in momentum space is then

$$\psi_{JLS}(q) = \int dr\, e^{-iq \cdot r}\Psi_{JLS}(r). \qquad (21)$$

Heavy-quarkonium bound states have been obtained previously with many different forms of potentials [18].

In nonrelativistic kinematics with a center-of-mass motion, $P = (P_0, P)$, and $q = (q_0, q) \approx (0, q)$. One can perform a decomposition of the relative wave function in terms of color and angular momentum states. The amplitudes in Eq. (7) can be decomposed as

$$\mathcal{M}(ab \to Q\bar{Q}x)|Q\bar{Q}\rangle = \sum_{cJLS}\mathcal{M}_{JLS}^c(qx)|qP\rangle, \qquad (22)$$

where $\mathcal{M}_{JLS}^c(qx)$ is

$$\mathcal{M}_{JLS}^c(qx) = \mathcal{A}_{JLS}^c(|q|x)\mathcal{Y}_{JLS}^c(\hat{q}) \qquad (23)$$

with $\mathcal{Y}_{JLS}^c(\hat{q})$ the angular momentum and color wave function. For simplicity of notation, the azimuthal angular momentum component M and the color component of the color multiplet in $\mathcal{Y}_{JLS}^c$ will not be written out explicitly. The bound state ψ_{JLS} can be described as

$$|\psi_{JLS}P\rangle = \int \frac{dq}{(2\pi)^3}\psi_{JLS}(q)|qP\rangle. \qquad (24)$$

In the lowest order of approximation without soft gluon emission, the probability amplitude for the direct production of ψ_{JLS} is obtained by projecting ψ_{JLS} onto $\Phi_{ab}(t_i)$. We use the normalization for the center-of-mass motion

$$\langle P'|P\rangle = (2\pi)^3 2E_P \delta(\boldsymbol{P}' - \boldsymbol{P}) \qquad (25)$$

and the usual convention for nonrelativistic relative motion

$$\langle \boldsymbol{q}'|\boldsymbol{q}\rangle = (2\pi)^3 \delta(\boldsymbol{q}' - \boldsymbol{q}). \qquad (26)$$

To carry out the projection of $\Phi_{ab}(t_i)$ onto ψ_{JLS}, we use Eq. (22) to write Eqs. (8) and (9) as

$$|\Phi_{ab}^{Q\bar{Q}}(t_i)\rangle = \int \frac{d^3Q}{(2\pi)^3 2E_Q} \frac{d^3\bar{Q}}{(2\pi)^3 2E_{\bar{Q}}} (2\pi)^4$$

$$\times \delta^{(4)}(P_i - P_f) \sum_{cJLS} \mathcal{M}_{JLS}^c(\boldsymbol{q})|\boldsymbol{q}P\rangle, \qquad (27)$$

$$|\Phi_{ab}^{Q\bar{Q}x}(t_i)\rangle = \int \frac{d^3Q}{(2\pi)^3 2E_Q} \frac{d^3\bar{Q}}{(2\pi)^3 2E_{\bar{Q}}} \frac{d^3g}{(2\pi)^3 2E_g}$$

$$\times (2\pi)^4 \delta^{(4)}(P_i - P_f) \sum_{cJLS} \mathcal{M}_{JLS}^c(\boldsymbol{q}x)|\boldsymbol{q}P\rangle. \qquad (28)$$

The scalar product of $|\psi_{JLS}Px\rangle$ with the above state vector, $\langle \psi_{JLS}Px|\Phi_{ab}^{Q\bar{Q}x}(t_i)\rangle$, can be evaluated by transforming to the variables P and q with Eq. (14). Using Eq. (24), we obtain

$$\langle \psi_{JLS}Px|\Phi_{ab}^{Q\bar{Q}x}(t_i)\rangle = (2\pi)^4 \delta^{(4)}(P_{ab} - P_\psi)\frac{2}{m_{JLS}}$$

$$\times \int \frac{d^3q}{(2\pi)^3} \psi_{JLS}^*(\boldsymbol{q})\mathcal{M}_{JLS}^{(1)}(\boldsymbol{q}x). \qquad (29)$$

Therefore, from the above result and the definition of $\mathcal{M}(ab \rightarrow \psi_{JLS}x)$ as given by Eq. (10), we get

$$\mathcal{M}(ab \rightarrow \psi_{JLS}x) = \frac{2}{m_\psi}\langle \psi_{JLS}(\boldsymbol{q})|\mathcal{M}_{JLS}^{(1)}(\boldsymbol{q}x)\rangle, \qquad (30)$$

where

$$\langle \psi_{JLS}(\boldsymbol{q})|\mathcal{M}_{JLS}^{(1)}(\boldsymbol{q}x)\rangle = \int \frac{d^3q}{(2\pi)^3} \psi_{JLS}^*(\boldsymbol{q})\mathcal{M}_{JLS}^{(1)}(\boldsymbol{q}x), \qquad (31)$$

which involves only color-singlet components of $|\Phi_{ab}(t_i)\rangle$. Thus, if the bound state wave function is known, the above overlap will give the matrix element and the color-singlet contribution to the quarkonium production cross section in Eq. (13).

In many approximate calculations one expands $\mathcal{A}_{JLS}^{(1)}(|\boldsymbol{q}|x)$ in powers of the velocity $v = |\boldsymbol{q}|/M_Q$ and retains only the leading term

$$\mathcal{A}_{JLS}^{(1)}(|\boldsymbol{q}|x) \sim \bar{\mathcal{A}}_{JLSx}^{(1)} \times \left|\frac{\boldsymbol{q}}{M_Q}\right|^L, \qquad (32)$$

where

$$\bar{\mathcal{A}}_{JLSx}^{(1)} = \frac{M_Q^L}{L!}\left[\frac{d^L}{d|\boldsymbol{q}|^L}\mathcal{A}_{JLS}^{(1)}(|\boldsymbol{q}|x)\right]_{|\boldsymbol{q}|\rightarrow 0}. \qquad (33)$$

Then the above projection in Eqs. (30) and (31) gives the Lth derivative of the wave function at the origin [19–21]:

$$\langle \psi_{JLS}(\boldsymbol{q})|\mathcal{M}_{JLS}^{(1)}(\boldsymbol{q}x)\rangle = \bar{\mathcal{A}}_{JLSx}^{(1)}\left\langle \psi_{JLS}(\boldsymbol{q})\left|\left|\frac{\boldsymbol{q}}{M_Q}\right|^L\right|\mathcal{Y}_{JLS}^c\right\rangle$$

$$= \bar{\mathcal{A}}_{JLSx}^{(1)}(-i)^L\frac{(2L+1)!!}{M_Q^L 4\pi L!}\left[\frac{d^L R_{JLS}(r)}{dr^L}\right]_{r\rightarrow 0}. \qquad (34)$$

The cross section can then be separated into a short-distance part involving $\bar{A}_{JLSx}^{(1)}$ and the long-distance nonperturbative part involving the Lth derivative of the bound state wave function at the origin. We can write the above results in the notation of the color-octet model of Eq. (1):

$$\sigma(\psi) = \sum_{ab}\sum_{c,J,L,S} F(ab \rightarrow {}^{2S+1}L_J^c)\frac{\langle 0|O({}^{2S+1}L_J^c \rightarrow \psi)|0\rangle}{M_Q^{2L}}, \qquad (35)$$

where

$$F(ab \rightarrow {}^{2S+1}L_J^{(1)})$$

$$= \int \frac{1}{4I}(2\pi)^4 \delta^{(4)}(P_i - P_f)$$

$$\times \left(\frac{2}{m_\psi}\right)^2 \sum_x \frac{L!|\bar{\mathcal{A}}_{JLSx}^{(1)}|^2}{4\pi 2N_c(2L+1)!!}d\Phi_{\psi x}, \qquad (36)$$

and

$$\frac{\langle 0|O(^{2S+1}L_J^{(1)}\to\psi)|0\rangle}{M_Q^{2L}}=\frac{2N_c}{4\pi}\frac{(2L+1)!!}{L!}\left[\frac{d^L R_{JLS}(r)}{M_Q^L dr^L}\right]^2_{r\to0},$$

(37)

which agrees with Bodwin et al. [1].

In addition to these color-singlet contributions, the color-octet model further postulates a much slower process with the emission of a soft gluon g_s, different from the emission of the hard gluon g in perturbative QCD already included in the second amplitude $|\Phi_{ab}^{Q\bar{Q}g}\rangle$ of Eq. (7). One can describe the emission of a soft gluon by studying further evolution of the state $|\Phi_{ab}(t_i)\rangle$ under the influence of the color field in the form

$$|\Phi_{ab}(t)\rangle = U(t,t_i)|\Phi_{ab}(t_i)\rangle$$

$$\sim\left(1-ig\int_{t_i}^t\int\hat{j}\cdot\hat{A}\ d^3x\ dt\right)|\Phi_{ab}(t_i)\rangle,\quad(38)$$

where we shall focus attention on soft gluons only for the gluon field A.

The probability amplitude $\mathcal{M}(ab\to\psi_{JLS}g_s)$ for the production of the bound state $\psi(JLS)$ accompanied by a soft gluon g_s is

$$(2\pi)^4\delta(P_{ab}-P_\psi-p_g)\mathcal{M}(ab\to\psi_{JLS}g_s)$$

$$=\langle(\psi_{JLS}P_\psi q)g_s|\Phi_{ab}(t)\rangle$$

$$=\langle(\psi_{JLS}P_\psi q)g_s|(-ig)\int\hat{j}\cdot\hat{A}\ d^3r\ dt|\Phi_{ab}(t_i)\rangle,$$

(39)

where we have used the notation g_s with the subscript s to denote a soft gluon emitted by a color-octet state. Using Eqs. (8) and (14), the right-hand side (RHS) of the above equation becomes

RHS of (39) $=\sum_{J'L'S'}\int\frac{d^4P}{(2\pi)^3}\delta(P^2-m_P^2)\frac{d^4q}{(2\pi)^3}\delta\left(\frac{P\cdot q}{2}\right)$

$$\times(2\pi)^4\delta^{(4)}(P_{ab}-P)\langle(\psi_{JLS}P_\psi q)g_s|(-ig)$$

$$\times\int_{t_i}^t\int\hat{j}\cdot\hat{A}\ d^3r\ dt\ \mathcal{M}_{J'L'S'}^{(8)}(q)|qP\rangle.\quad(40)$$

One envisages that in order to emit a soft gluon, the state Φ_{ab} must have evolved for a long time and has characteristics of nonperturbative states. It is therefore reasonable to postulate that the soft gluon is emitted by an intermediate nonperturbative state $\psi_{J'L'S'}^{(8)}$ best described nonperturbatively in terms of Q and $\bar{Q}$ interacting in a potential. It is appropriate to expand the intermediate states in terms of $\psi_{J'L'S'}^{(8)}$ with an invariant mass $m_{J'L'S'}$ using

$$\sum_{J'L'S'}\int dm_{J'L'S'}\frac{dn}{dm_{J'L'S'}}|\psi_{J'L'S'}^{(8)}\rangle\langle\psi_{J'L'S'}^{(8)}|=1.$$

(41)

Here $dn/dm_{J'L'S'}$ is the density of these color-octet $\psi_{J'L'S'}^{(8)}$ states with a specific polarization and color-octet component

$$\frac{dn}{dm_{J'L'S'}}=\frac{m_Q^{3/2}}{(2\pi)^2}\sqrt{m_{J'L'S'}-2m_Q-\frac{L'(L'+1)}{m_Q R^2}},$$

(42)

where R is the average radius of the Q-$\bar{Q}$ separation of the color-octet state which emits the soft gluon. It can be taken approximately as the Q-$\bar{Q}$ separation of the final color-singlet quarkonium state. For the emission of a soft gluon through the intermediate color-octet state $\psi_{J'L'S'}^{(8)}$, the right-hand side of Eq. (39) is

RHS of Eq. (39) $=\sum_{J'L'S'}\int\frac{d^4P}{(2\pi)^3}\delta(P^2-m_{J'L'S'}^2)\frac{d^4q}{(2\pi)^3}\delta\left(\frac{P\cdot q}{2}\right)(2\pi)^4\delta(P_{ab}-P)$

$$\times\langle(\psi_{JLS}P_\psi)g_s|(-ig)\int_{t_i}^t\int\hat{j}\cdot\hat{A}\ d^3x\ dt|\psi_{J'L'S'}^{(8)},P\rangle dm_{J'L'S'}\frac{dn}{dm_{J'L'S'}}\langle\psi_{J'L'S'}^{(8)}|\mathcal{M}_{J'L'S'}^{(8)}(q)|q\rangle.$$

(43)

The integral over q can be carried out to give

$$\int\frac{d^4q}{(2\pi)^3}\delta\left(\frac{P\cdot q}{2}\right)\langle\psi_{J'L'S'}^{(8)}|\mathcal{M}_{J'L'S'}^{(8)}(q)|q\rangle=\frac{2}{m_{J'L'S'}}\int\frac{d^3q}{(2\pi)^3}\psi_{J'L'S'}^{(8)*}(q)\mathcal{M}_{J'L'S'}^{(8)}(q)\equiv\frac{2}{m_{J'L'S'}}\langle\psi_{J'L'S'}^{(8)}(\mathbf{q})|\mathcal{M}_{J'L'S'x}^{(8)}(\mathbf{q})\rangle.$$

(44)

Thus, the amplitude involves a projection of the color-octet Feynman amplitude onto a color-octet nonperturbative amplitude,

$\langle \psi_{J'L'S'}^{(8)}(q)|\mathcal{M}_{J'L'S'}^{(8)}(q)\rangle$, similar to the projection of the color-singlet component onto a color-singlet amplitude as given in Eq. (30). One can thus similarly relate this projection to the Lth radial derivative of the spatial wave function, as was shown in Eq. (34). One obtains

$$\frac{\langle \psi_{J'L'S'}^{(8)}(q)|\mathcal{M}_{J'L'S'}^{(8)}(q)\rangle}{M_Q^{L'}} = \bar{\mathcal{A}}_{J'L'S'}^{(8)}(-i)^{L'}\frac{(2L'+1)!!}{M_Q^{L'}4\pi L'!}\left[\frac{d^{L'}R_{J'L'S'}(r)}{dr^{L'}}\right]_{r\rightarrow 0}, \tag{45}$$

where

$$\bar{\mathcal{A}}_{J'L'S'}^{(8)} = \frac{M_Q^L}{L'!}\left[\frac{d^{L'}}{d|q|^{L'}}\mathcal{A}_{J'L'S'}^{(8)}(|q|)\right]_{|q|\rightarrow 0}. \tag{46}$$

To evaluate the long-distance soft gluon radiation matrix element, we have

$$\hat{A}^\mu = \sum_k \sqrt{\frac{4\pi}{2\omega_g}}[\hat{c}_k\epsilon^\mu e^{i(k\cdot r-\omega_g t)}+\hat{c}_k^\dagger\epsilon^{\mu*}e^{-i(k\cdot r-\omega_g t)}], \tag{47}$$

where ω_g is the soft gluon energy, ϵ^μ is the gluon polarization vector, c_k and $c_k^\dagger$ are the creation and annihilation operator for the soft gluon. Therefore, we have

$$\langle(\psi_{JLS}P_\psi)g_s(p_g)|(-ig)\int_{t_i}^t\hat{j}\cdot\hat{A}\ d^3r\ dt|\psi_{J'L'S'}^{(8)}(q)P\rangle$$

$$= V_{J'L'S'\rightarrow JLS}(2\pi)^3\delta(P-P_\psi-P_g)$$

$$\times\int_{t_i}^t e^{-i(E_i-E_f-\omega_g)t}dt,$$

where

$$V_{J'L'S'\rightarrow JLS} = -ig\sqrt{\frac{4\pi}{2\omega_g}}\epsilon_\mu^*\int j_{J'L'S'\rightarrow JLS}^\mu(r)e^{-ik\cdot r}\ dr, \tag{48}$$

the timelike component of $j_{fi}^0(r)=\rho_{fi}(r)$ is

$$\rho_{fi}(r) = \int dr_1\ dr_2\ \psi_f^*(r_1,r_2)\sum_{n=1}^2\ \delta(r-r_n)\psi_i(r_1,r_2), \tag{49}$$

where r_1, r_2 refer to the coordinates of the heavy quark and antiquark, and ψ_i and ψ_f are the initial and final wave functions. The spacelike component is

$$j_{fi}(r) = -\frac{ig}{2M_Q}[\psi_f^*(r)\nabla\psi_i(r)-\psi_i^*(r)\nabla\psi_f(r)]$$

$$+\frac{g\mu_Q}{2M_Q}\nabla\times[\psi_f^*(r)\sigma\psi_i(r)], \tag{50}$$

where $\mu_Q\sim 2$ is the color-magnetic moment of the quark in units of the color Bohr magneton. Therefore, from Eqs. (39) and (43), we have

$$\mathcal{M}(ab\rightarrow\psi_{J'L'S'}^{(8)}\rightarrow\psi_{JLS}g_s)$$

$$= \langle\psi_{J'L'S'}^{(8)}(q)|\mathcal{M}_{J'L'S'}^{(8)}(q)\rangle V_{J'L'S'\rightarrow JLS}$$

$$\times\frac{\sqrt{2E_{J'L'S'}}}{m_{J'L'S'}^2}2\pi\frac{dn}{dm_{J'L'S'}}, \tag{51}$$

where $E_{J'L'S'}=\sqrt{m_{J'L'S'}^2+(a+b)^2}$. From Eq. (13)

$$d\sigma(ab\rightarrow\psi_{JLS}g_s)$$

$$= \frac{1}{4I}\sum_{xJ'L'S'}|\mathcal{M}(ab\rightarrow\psi_{J'L'S'}^{(8)}\rightarrow\psi_{JLS}g_s)|^2(2\pi)^4$$

$$\times\delta^{(4)}(P_i-P_f)d\Phi_{\psi g_s}, \tag{52}$$

where the phase space volume element $d\Phi_{\psi g_s}$ includes both the soft and the hard gluons. For this case when only a soft gluon is emitted, $d\Phi_{\psi g_s}$ is given by

$$d\Phi_{\psi g_s} = \frac{d^3P_\psi}{(2\pi)^32E_\psi}\frac{d^3g_s}{(2\pi)^32E_{g_s}}. \tag{53}$$

We have then

$$d\sigma(ab\rightarrow\psi_{JLS}g_s)$$

$$= \frac{1}{4I}\sum_{xJ'L'S'}\left(\frac{2}{m_{J'L'S'}}\right)^2|\langle\psi_{J'L'S'}^{(8)}(q)|\mathcal{M}_{J'L'S'}^{(8)}(q)\rangle|^2$$

$$\times|V_{J'L'S'\rightarrow JLS}|^2\frac{1}{4m_{J'L'S'}^2}\left(2\pi\frac{dn}{dm_{J'L'S'}}\right)^2$$

$$\times\frac{|p_\psi|}{8\pi^2}d\Omega_\psi, \tag{54}$$

where $|\boldsymbol{p}_\psi|$ is the magnitude of the momentum of ψ_{JLS} in the rest frame of $\psi^{(8)}_{J'L'S'}$. One can expand $\langle \psi^{(8)}_{J'L'S'}(\boldsymbol{q}) | \mathcal{M}^{(8)}_{J'L'S'}(\boldsymbol{q}) \rangle$ out according to Eq. (45) and obtain

$$
\begin{aligned}
&d\sigma(ab \rightarrow \psi_{JLS} g_s) \\
&= \frac{1}{4I} \sum_{xJ'L'S'} \left(\frac{2}{m_{J'L'S'}} \right)^2 |\bar{\mathcal{A}}^{(8)}_{J'L'S'}|^2 \\
&\quad \times \frac{(2L'+1)!!}{4\pi L'!} \left| \left[\frac{d^{L'} R_{J'L'S'}(r)}{M_Q^{L'} dr^{L'}} \right]_{r \rightarrow 0} \right|^2 \\
&\quad \times |V_{J'L'S' \rightarrow JLS}|^2 \frac{1}{8m^2_{J'L'S'}} \left(\frac{dn}{dm_{J'L'S'}} \right)^2 |\boldsymbol{p}_\psi| d\Omega_\psi .
\end{aligned}
\tag{55}
$$

We can then compare the above results with the expressions in the color-octet model, which we write in the form

$$
\begin{aligned}
&d\sigma(ab \rightarrow \psi_{JLS} g_s) \\
&= \frac{1}{4I} \sum_{J'L'S'} f(ab \rightarrow J'L'S')(2\pi)^4 \\
&\quad \times \delta^{(4)}(p_a + p_b - p_\psi) \frac{d^3 p_\psi}{(2\pi)^3 2E_\psi} \frac{\langle O_8^{JLS}(J'L'S') \rangle}{M_Q^{2L'}} ,
\end{aligned}
\tag{56}
$$

where

$$
\begin{aligned}
&f(ab \rightarrow J'L'S') \\
&= \frac{1}{4I} \sum_{J'L'S'} \left(\frac{2}{m_{J'L'S'}} \right)^2 \left| \sqrt{\frac{(2L'+1)!!}{4\pi C_{J'L'S'} L'!}} \bar{\mathcal{A}}^{(8)}_{J'L'S'} \right|^2 ,
\end{aligned}
\tag{57}
$$

where $C_{J'L'S'} = (\text{number of polarization}) \times (N_c^2 - 1)$ for the color-octet state $\{J'L'S'\}$. We observe the following differences. The color-octet model result of Eq. (56) assumes that the soft photon does not carry energy, hence the cross section is a delta function of the total parton invariant mass, centered at the invariant mass of ψ_{JLS}. The soft gluon makes no contribution to the phase space volume element and the matrix element is parametrized as unknown constants. On the other hand, when one takes into account the details of the emission process to obtain the results in Eq. (55), the cross section is a continuous function of the total parton invariant mass $m_{J'L'S'}$. The production of ψ_{JLS} occurs within the invariant mass interval from the minimum $m_{J'L'S'}(\min)$, at which the color-octet state begins to exist, to the maximum value $m_{J'L'S'}(\max) = m_{JLS} + \omega_g(\text{cutoff})$, which corresponds to the maximum gluon energy of the soft gluon process. One envisages that gluons with energies ω_g higher than $\omega_g(\text{cutoff})$ will be emitted at a shorter time scale and will be described by perturbative QCD and not by the soft gluon emission process through an intermediary nonperturbative state. As the time to travel the length of $R/2$ of a J/ψ corresponds to an energy scale of $2\hbar/R$, this $\omega_g(\text{cutoff})$ energy should be smaller for a final quarkonium state with a greater radius (such as ψ'). Furthermore, another difference is that the cross section of Eqs. (52) and (55) in the present description is given in terms of transition matrix elements of nonperturbative wave functions, and the phase-space volume element includes the effect of the additional soft gluon.

In an approximate comparison by integrating Eqs. (55) and (56) over $m_{J'L'S'}$, we get the approximate equivalence

$$
\begin{aligned}
\frac{\langle O^{(8)}(J'L'S' \rightarrow JLS) \rangle}{M_Q^{2L'}} &\sim E_{JLS} \int_{m_{J'L'S'}(\min)}^{m_{J'L'S'}(\max)} dm_{J'L'S'} \left| \sqrt{\frac{C_{J'L'S'}(2L'+1)!!}{4\pi L'!}} \left[\frac{d^{L'} R_{J'L'S'}(r)}{M_Q^{L'} dr^{L'}} \right]_{r \rightarrow 0} \right|^2 \\
&\quad \times |V_{J'L'S' \rightarrow JLS}|^2 \frac{1}{8\pi m^2_{J'L'S'}} \left(\frac{dn}{dm_{J'L'S'}} \right)^2 |\boldsymbol{p}_\psi| d\Omega_\psi .
\end{aligned}
\tag{58}
$$

We shall consider the soft gluon to have large wave lengths which are greater than the dimension of the radiating system. In this long wavelength approximation the matrix element V_{fi} has been worked out in detail [see Eqs. (46.6) and (46.7) of Ref. [22]]. The results for gluon radiation of multipolarity LM are

$$
|V_{fi}|^2 = 4 \frac{(L+1)}{L} \frac{\omega_g^{2L+1}}{[(2L+1)!!]^2} |M_{LM}^{(e,m)}|^2 .
\tag{59}
$$

For electric transition of multipolarity LM, the multipole transition moment is

$$
M_{LM}^{(e)} = g \int d\boldsymbol{r} \, \rho_{fi}(\boldsymbol{r}) r^L Y_{LM}(\hat{\boldsymbol{r}}) .
\tag{60}
$$

For magnetic transition of multipolarity LM, the multipole transition moment is

$$
M_{LM}^{(m)} = \frac{1}{L+1} \int d\boldsymbol{r} \, \boldsymbol{r} \times \boldsymbol{j}_{fi}(\boldsymbol{r}) \cdot \nabla (r^L Y_{LM}(\hat{\boldsymbol{r}})) .
\tag{61}
$$

TABLE I. Charmonium states obtained with the screened potential Eq. (63).

Bound state	Resonance	Calculated energy (GeV)	Experimental energy (GeV)	Calculated Γ_{ee^+} (keV)	Experimental Γ_{ee^+} (keV)
1S		3.145	3.07	8.11	5.26±0.37
2S		3.560	3.66	2.57	2.14±0.21
	3S	3.947	4.04		
1P		3.476	3.52		
	2P	3.971			
	1D	3.801	3.77		

III. POTENTIAL MODEL FOR Q AND $\bar{Q}$ INTERACTING AT LARGE DISTANCES

We wish to describe the nonperturbative interaction between Q and $\bar{Q}$ in terms of a potential model. Such an interaction is known for Q and $\bar{Q}$ in the color-singlet state. The interaction consists of an attractive short-range color-Coulomb interaction and a long-range linear potential. For Q and $\bar{Q}$ in a color-octet state, the short-range color-Coulomb interaction is repulsive. From lattice calculations for gluonic excitation in the presence of a static quark-antiquark pair, the spectrum appears to rise linearly with the relative separation between Q and $\bar{Q}$, indicating an effective linear potential for color-octet states [12]. We can write the Q-$\bar{Q}$ potential as

$$V_c(r) = C_c \frac{\alpha_s}{r} + \sigma_c r \quad (c=1,8), \qquad (62)$$

where $C_1 = -4/3$, $C_8 = 1/6$, α_s is the strong interaction coupling constant, $\sigma_1 \sim 1$ GeV/fm, but σ_8 is not known.

The confining potential given in Eq. (62) is a good description if the spontaneous production of light quark pairs is not considered. However, when a quark and an antiquark pull far apart, the string joining Q and $\bar{Q}$ breaks. Evidence of string breaking comes experimentally in the production of a large number of pions in e^+e^- annihilation at high energies [23,24]. Theoretically, the pair production will occur spontaneously through the Schwinger mechanism when Q is separated from $\bar{Q}$ at a distance r such that σr is greater than the mass of a $q\bar{q}$ pair [25]. When the string breaks, the interaction between Q and $\bar{Q}$ will be screened and will become a weak color–van del Waal–type interaction, instead of the strong linear interaction. The asymptotic behavior of the Q-$\bar{Q}$ potential is therefore drastically changed when the spontaneous production of light quark pairs is allowed. Instead of the confining behavior as usually assumed, it is more appropriate to describe the motion of the Q and $\bar{Q}$ effectively as having an asymptotically free motion at $r \to \infty$, on account of the breakup of the string due to the spontaneous production of a light quark pairs $q\bar{q}$, leading to the production of a $Q\bar{q}$ and a $\bar{Q}q$.

The influence of the dynamical light quarks on the Q-$\bar{Q}$ potential has been investigated theoretically by Laermann and collaborators [26,27] using lattice gauge theory. It is found that including the dynamical quarks in the lattice gauge theory leads to a potential which deviates from the linear potential. The modified interaction can be described in the form of a screened color-Yukawa potential. Because of this screening due to dynamical quarks, a more realistic Q-$\bar{Q}$ potential becomes [26,27]

$$V_c(r) = C_c \frac{\alpha_s e^{-\mu r}}{r} + \sigma_c r \frac{1 - e^{-\mu r}}{\mu_c r} - \frac{\sigma_c}{\mu_c}, \qquad (63)$$

where we have used the reference that the potential vanishes at $r \to \infty$.

Under the potential $V_c(r)$ given above, nonperturbative $Q\bar{Q}$ states $\psi_{JLS}^c(r)$ can be obtained by solving the Schrödinger equation

$$\left\{ -\frac{1}{M_Q} \nabla^2 + V_c(r) \right\} \psi_{JLS}^c(\boldsymbol{r}) = \epsilon \psi_{JLS}^c(\boldsymbol{r}), \qquad (64)$$

where $\epsilon = m_{JLS} - 2M_D$.

In our approximate treatment for charmonium, we consider a single heavy object which can travel at all distances, with the provision that in going from small distances to large distances, the nature of this object changes from being a charm quark at small distances to becoming an open charm D-meson at large distances. Thus, the mass of this heavy object should also change from that of the charm quark mass to a D-meson mass. We describe such a variation of the mass by the effective mass

$$M_Q(r) = M_D - \frac{M_D - M_c}{1 + \exp\{(r - r_m)/a_m\}}, \qquad (65)$$

where we shall use $r_m = 0.8$ fm and $a_m = 0.2$ fm to describe the change in nature of this heavy object at a distance of $r_m \sim 0.8$ fm.

We use the potential (63) and the effective mass (65) to study the charmonium bound states and the resonances in the continuum. The latter are characterized by an asymptotic phase shift of $(2n+1)\pi/2$ where n is an integer. A reason-

able description of the bound states and resonances can be obtained (Table I) by using the parameters of $\alpha_s = 0.3$, $\sigma_1 = 1.7$ GeV/fm, $\mu = 0.28$ GeV, and $M_c = 1.6$ GeV. A potential similar to Eq. (63) has been used successfully to study the energy spectrum of charmonium states [28]. As in this earlier work, we find that a large value of the parameter σ_1 is needed when the linear interaction is modified to take the form of the screened potential of Eq. (63). We shall use this potential to generate the J/ψ and ψ' wave functions for subsequent studies.

The behavior of the color-octet $(Q\bar{Q})_8$ states can be studied by looking at the states in the potential (63). The color-octet potential is repulsive at short distances. For sufficiently large values of σ_8, there is a potential pocket at intermediate distances. Depending on the mass of the quark and the string tension σ_8, the pocket of the color-octet potential may hold a bound state. The greater the rest mass of the quark, the greater the chance of a color-octet bound state. In this respect, there may be a greater chance of finding a color-octet bound state in a $b\bar{b}$ system compared to a $c\bar{c}$ system.

If the linear potential parameter σ_8 for color-octet states is greater than 0.31 GeV/fm, then the potential pocket will hold a bound $[c\bar{c}]_8$ state. (The state is bound in the sense that its mass is less than $2M_D$, but it is unstable against the emission of a gluon.) There is, however, little experimental information on the color-octet states of either a $c\bar{c}$ or a $b\bar{b}$ system. Compared to color-octet states in the continuum, a bound color-octet state is distinguished by its relatively longer lifetime, as the only decay mode is the emission of soft gluons, and the emission rate is lower for the bound state because the energy difference between the initial and the final state is less for a bound color-octet state. A long-lived color-octet quarkonium will travel a large distance before its color is neutralized.

It is of interest to examine the signature of a long-lived bound color octet state. Its only decay channel is the emission of a gluon to come down to color-singlet $Q\bar{Q}$ bound states. The emitted gluon will be converted to a light $q\bar{q}$ pair. If the initial color-octet $c\bar{c}$ pair has a high kinetic energy relative to its complementary color-octet partner (which is either a $[q\bar{q}]_8$ or a $[q(qq)]_8$), then the $q\bar{q}$ from the emitted gluon will form two chains of light mesons (mostly pions) with the color-octet partner as shown in Fig. 1.

In the more general case when the color-octet is accompanied by two color-octet partners (one from the projectile hadron and one from the target hadron), the emitted gluon from the $[Q\bar{Q}]_8$ color-octet needs to branch out into two gluons, which will be converted into two pairs of light $[q\bar{q}]_8$ pairs to form four chains of produced light mesons.

To search for long-lived color-octet states, it is useful to look at the kinematic regions where the produced heavy quarkonium is kinematically separated from the other debris of the collision, as in the case of J/ψ or Υ at large values of x_F. The signature of a long-lived color-octet quarkonium state may show up as a quarkonium at one end accompanied by hadrons which arise from the breaking of strings joining this color-octet and its complementary color object at the
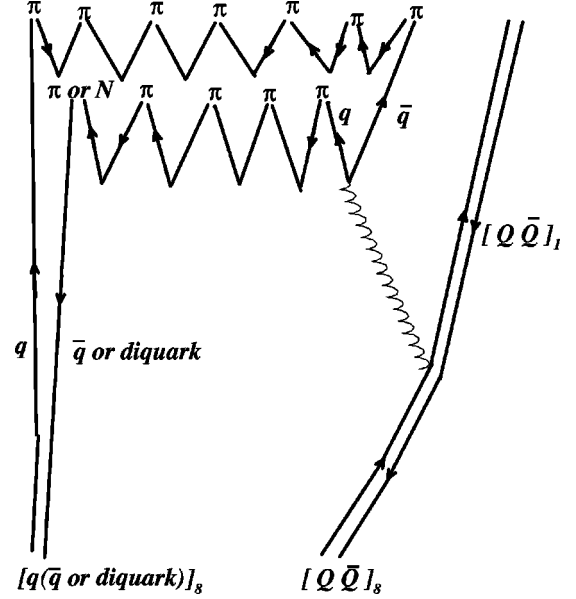

FIG. 1. Schematic picture to show how a bound color-octet $[Q\bar{Q}]_8$ and its color-octet partner $[q\bar{q}]_8$ (or $[q(qq)]_8$) evolve into color-singlet objects.

other end. There will be chains of pions in the rapidity gap between the produced quarkonium state and its complementary partner. Thus, the search and the measurement of the chains of hadrons with a leading quarkonium state may reveal the existence and the characteristics of the quarkonium as a color-octet state. In the situation that the pocket is not deep enough to hold a bound color-octet state, then the intermediate color-octet state $\psi^{(8)}_{J'L'S'}$, through which the soft gluon is emitted, can only be in the continuum.

IV. CONTINUUM STATES

For an intermediate color-octet state in the continuum, the cross section for the emission of soft gluons depends on its wave function in two ways. First, it is needed to give the overlap between the amplitude from perturbative QCD as given by Eq. (45). It is also needed to obtain the transition matrix element for soft gluon emission (given in the next section).

To obtain the continuum wave function, we follow the phase-angle method discussed in detail by Calogero [29]. We write the wave function as

$$\psi_{JLS}(\boldsymbol{r}) = R_{JLS}(r)\mathcal{Y}_{JLS}(\hat{\boldsymbol{r}}) = \frac{u_{JLS}(r)}{kr}\mathcal{Y}_{JLS}(\hat{\boldsymbol{r}}) \qquad (66)$$

and represent the wave function in terms of the amplitude $\alpha_L(r)$ and the phase shift $\delta_L(r)$

$$u_{JLS}(r) = \frac{\alpha_L(r)}{\alpha_L(\infty)}\hat{D}_L(kr)\sin[\hat{\delta}_L(kr) + \delta_L(r)]\sqrt{\frac{4\pi}{2l+1}}, \qquad (67)$$

with the boundary condition that $\delta_L(r \to 0) = 0$. The functions $\hat{D}(kr)$ and $\hat{\delta}(kr)$ are known functions [29]:

$$\hat{D}_0(x) = 1, \quad \hat{D}_1(x) = (1 + 1/x^2)^{1/2} \quad (68)$$

and

$$\hat{\delta}_0(x) = x; \quad \hat{\delta}_1(x) = x - \tan^{-1} x. \quad (69)$$

The equation for $\delta_L(r)$ is [29]

$$\frac{d}{dr} \delta_L(r) = -\frac{U(r)}{k} \hat{D}_L^2(kr)\{\sin[\hat{\delta}_L(kr) + \delta_L(r)]\}^2, \quad (70)$$

where $U(r) = M_Q(r) V_c(r) + [M_D - M_Q(r)]\epsilon$. After the function $\delta_L(r)$ is evaluated, the amplitude can be obtained from $\delta_L(r)$ by

$$\alpha_L(r) = \exp\left\{\frac{1}{2k} \int_0^r ds\, U(s) \hat{D}_L^2(ks) \sin 2[\hat{\delta}_L(ks) + \delta_L(s)]\right\}. \quad (71)$$

The numerical integration of Eq. (70) gives the asymptotic phase shift $\delta(\infty)$ and the continuum wave function. The energies at which the asymptotic phase shifts $\delta(\infty)$ are $(2 \times \text{integer} + 1)\pi/2$ are the locations of the resonances as shown in Table I.

For the wave function and its derivatives at the origin, it is useful to express the effect of the potential in terms of the K factor, the analogue of the Gamow factor for the Coulomb interaction [30]. The wave function [Eq. (67)] near the vicinity of the origin is given by

$$R_{JLS}(r) = \frac{u_{JLS}(r)}{kr} = \frac{\alpha_L(r)}{\alpha_L(\infty)} j_L(kr) \sqrt{\frac{4\pi}{2L+1}}$$

$$\sim \frac{\alpha_L(r)}{\alpha_L(\infty)} \frac{(kr)^L}{(2L+1)!!} \sqrt{\frac{4\pi}{2L+1}}. \quad (72)$$

Therefore, for the factor in Eq. (55), we have

$$\sqrt{\frac{(2L+1)!!}{4\pi L!}} \frac{1}{M_Q^L} \frac{d^L R_{JLS}(r)}{dr^L}\bigg|_{r \to 0}$$

$$= \frac{\alpha_L(r)}{\alpha_L(\infty)} \frac{k^L}{M_Q^L} \sqrt{\frac{L!}{(2L+1)!!(2L+1)}}. \quad (73)$$

Compared to the case of no interaction for which $\alpha(r) = 1$, the probability is modified by a factor, the K factor,

$$K_L = \left|\frac{\alpha_L(r \to 0)}{\alpha_L(\infty)}\right|^2. \quad (74)$$

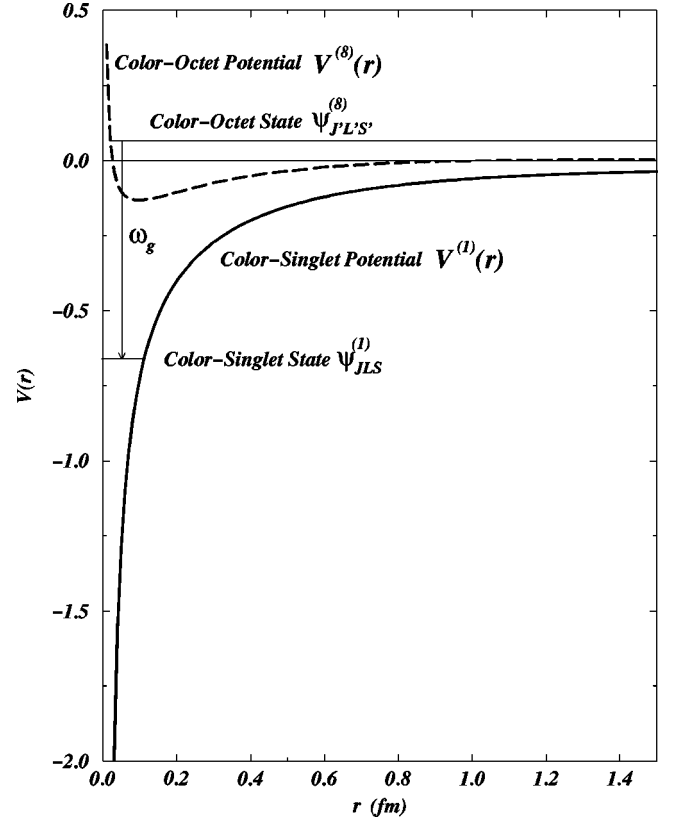In terms of the K factor, the square of the Lth derivative of the wave function at the origin is

FIG. 2. Schematic picture of the color-singlet potential $V^{(1)}(r)$, color-octet potential $V^{(8)}(r)$, the initial color-octet state $\psi^{(8)}_{J'L'S'}$, and the final color-singlet state $\psi^{(1)}_{JLS}$ in a soft-gluon emission process.

$$\left|\sqrt{\frac{(2L+1)!!}{4\pi L!}} \frac{1}{M_Q^L} \frac{d^L R_{JLS}(r)}{dr^L}\bigg|_{r \to 0}\right|^2$$

$$= K_L \left|\frac{k}{M_Q}\right|^{2L} \frac{L!}{(2L+1)!!(2L+1)}. \quad (75)$$

An example of the K factor is the Gamow factor arising from the Coulomb interaction. Our K factors obtained numerically can be checked by noting that for the Coulomb interaction with $V(r) = -\alpha/r$, the K factor can be obtained analytically and shown to be

$$K_L(\eta) = \frac{(L^2 + \eta^2)[(L-1)^2 + \eta^2] \dots (1 + \eta^2)}{[L!(2L+1)!!]^2}$$

$$\times \frac{2\pi\eta}{1 - \exp\{-2\pi\eta\}}, \quad (76)$$

where $\eta = \alpha/v$.

V. MULTIPOLE TRANSITIONS

We can evaluate the square of the matrix element $|V_{fi}|^2$ for the emission of soft gluons based on the analogous multipole expansion for the emission of electromagnetic waves [22,31].

We consider the production of a $Q\bar{Q}$ pair by parton collisions at an energy ϵ above the $D\bar{D}$ threshold. The $Q\bar{Q}$ can be in many different angular momentum states. After a nonperturbative time scale, the $Q\bar{Q}$ pair will evolve into nonperturbative states $\psi^{(8)}_{J'L'S'}$. We consider the continuum color-octet states $\psi^{(8)}_{J'=0,L'=0,S'=0}$ ($^1S_0^{(8)}$) and $\psi^{(8)}_{J,L'=1,S'=1}$ ($^3P_J^{(8)}$) which can decay down to the color-singlet $\psi^{(1)}_{J=1,L=0,S=1}$ ($^3S_1^{(1)}$) state by the emission of a gluon of energy $\omega_g=\epsilon+B$ where B is the binding energy of $^3S_1^{(1)}$ (see Fig. 2). (For brevity of notation, the color state superscript in the wave function ψ^c_{JLS} is often understood.) The square of the matrix elements for the emission of a gluon of energy $\omega_g=\epsilon+B$ from the continuum color-octet $Q\bar{Q}(^1S_0^{(8)})$ state to the bound color-singlet $H=(^3S_1^{(1)},S_z=0)$ state through an $M1$ transition is given by [22,31]

$$|V^{M1}(^1S_0^{(8)}\to {}^3S_1^{(1)})|^2=\left(\frac{8}{9}\right)\omega_g^3|M'_{10}|^2. \tag{77}$$

The magnetic dipole transition matrix element M'_{10} is

$$M'_{10}=\frac{g}{2M_Q}\sqrt{\frac{3}{4\pi}}\sum_{i=Q,\bar{Q}}\int d^3r\ \psi^{\dagger}_{Q\bar{Q}}(^1S_0^{(8)})\mu_i\sigma_{zi}\psi_H(^3S_1^{(1)}), \tag{78}$$

where μ_Q is the color-magnetic dipole moment of Q and $\mu_{\bar{Q}}=-\mu_Q\sim 2$. The $\psi_{Q\bar{Q}}$ wave function for the color-octet $Q\bar{Q}(J'=0,L'=0,S'=0)$ pair in the continuum can be written as

$$\psi_{Q\bar{Q}}(J'L'S')=\frac{u(^1S_0{}^{(8)},r)}{kr}\mathcal{Y}_{J'L'S'}. \tag{79}$$

The wave function for the bound ψ_{JLS} state can be written as

$$\psi_{JLS}=\frac{u(^3S_1^{(1)},r)}{r}\mathcal{Y}_{JLS}. \tag{80}$$

From Eqs. (77)–(80), the matrix element M'_{10} is given by

$$M'_{10}=\frac{2g}{M_Qk}\sqrt{\frac{3}{4\pi}}\int u(^1S_0{}^{(8)},r)u(^3S_1^{(1)},r)dr. \tag{81}$$

We shall evaluate the above matrix element with wave functions obtained from the potential model. As an illustration of the order of magnitude, we can consider the simple case of

$$u(^1S_0{}^{(8)},r)=\sin(kr)\sqrt{4\pi} \tag{82}$$

and the lowest bound $^3S_1$ state wave function as

$$\frac{u(^3S_1{}^{(1)},r)}{r}=2\sqrt{\kappa^3}e^{-\kappa r} \tag{83}$$

where $\kappa=\sqrt{M_QB}$. Then the square of the matrix element for $M1$ gluon radiation from $^1S_0^{(8)}$ to $^3S_1^{(1)}$ is

$$|V^{M1}(^1S_0^{(8)}\to {}^3S_1^{(1)})|^2=\frac{32g^2(\mu_Q-\mu_{\bar{Q}})^2\omega_g^3}{3M_Q^2}\frac{\kappa^5}{(k^2+\kappa^2)^4}. \tag{84}$$

The $M1$ transition rate is constant as $k\to 0$ because it involves the spatial overlap of two S-state wave functions.

The square of the matrix element for the emission of a gluon of energy ω_g from the continuum $[Q\bar{Q}(^3P_J^{(8)}),J_z=S_z]$ state to the bound $H=(^3S_1^{(1)},S_z)$ state through an $E1(m=0)$ transition is [31]

$$|V^{E1}(^3P_J^{(8)}\to {}^3S_1^{(1)})|^2=\left(\frac{8}{9}\right)\omega_g^3|Q_{10}|^2. \tag{85}$$

The electric dipole transition matrix element is

$$Q_{10}=g\sqrt{\frac{3}{4\pi}}\sum_{i=Q,\bar{Q}}\int d^3r\ \Psi^{\dagger}_{Q\bar{Q}}(^3P_J^{(8)})z_i\Psi_H(^3S_1^{(1)})$$

$$=\frac{ig}{k}\sqrt{\frac{3}{4\pi}}\int u(^3P_J^{(8)},r)ru(^3S_1^{(1)},r)dr. \tag{86}$$

As an illustration, we shall again evaluate this matrix element with a wave function for the noninteracting case where the $u(^3P_J^{(8)},r)$ is

$$u(^3P_J^{(8)},r)=\left(\frac{\sin kr}{kr}-\cos kr\right)\sqrt{\frac{4\pi}{3}}. \tag{87}$$

The square of the matrix element for $E1$ gluon radiation from $^3P_J^{(8)}$ to $^3S_1^{(1)}$ is

$$|V^{E1}(^3P_J^{(8)}\to {}^3S_1^{(1)})|^2=\frac{8\times 256g^2}{27}\omega_g^3\frac{\kappa^5k^2}{(k^2+\kappa^2)^6}. \tag{88}$$

The $E1$ transition rate vanishes as $k\to 0$. This arises because the transition matrix element involves the product of the S-state wave function, which peaks at the origin, with the continuum P-state wave function, which vanishes at the origin. The product is zero in the long wavelength ($k\to 0$) limit and increases with increasing kinetic energy.

For the simple case when there is no interaction in the color-octet state, the ratio of the squares of the matrix elements for the production of the lowest ψ state are

$$\frac{|V^{M1}(^1S_0^{(8)}\to {}^3S_1^{(1)})|^2}{|V^{E1}(^3P_J^{(8)}\to {}^3S_1^{(1)})|^2}=\frac{9\mu_Q^2}{16}\frac{\omega_g^2}{\epsilon M_Q}. \tag{89}$$

With $\mu_Q=2$, we see that this $M1/E1$ ratio is large for soft gluons which carry energies slightly greater than the binding energy B. The $M1$ transition arises from the spin transition current. In contrast, the $M1$ transition arising from the spatial transition currents would lead to an $M1/E1$ ratio varying as $v^2=4q^2/M_Q^2$. In the present case, because of the angular momentum and spin selection rules, there is only a contribution to the $M1$ transition from the spin transition current, and the usual velocity counting rule based on the spatial transi-

tion currents breaks down [31]. The ratio of the $|V_{ij}|^2$ for the production of the ψ' state will be more complicated as the ψ' wave function contains one node and the radial matrix elements will involve cancellation of contributions of opposite signs.

VI. CONTRIBUTIONS FROM $M1$ AND $E1$ TRANSITIONS TO J/ψ AND ψ' PRODUCTION

From Eq. (55), we note that the cross section for going through different intermediate color-octet states $J'L'S'$ depends mainly on the product

$$\left| C_{J'L'S'} \sqrt{\frac{(2L'+1)!!}{4\pi L'!}} \frac{1}{M_Q^{L'}} \left[\frac{d^{L'} R_{J'L'S'}(r)}{dr^{L'}} \right]_{r\to 0} \right|^2$$

$$\times |V_{J'L'S'\to JLS}|^2 \left(\frac{dn}{dm_{J'L'S'}} \right)^2 . \quad (90)$$

We can consider the production of the $J/\psi(^3S_1)$ state via an $M1$ transition from the intermediate color-octet state $\{J'L'S'\}=^1S_0$. There can also be an $E1$ transition from the intermediate color-octet state $\{J'L'S'\}=^3P_0$. To evaluate various quantities in the above equation, we need a description of the color-octet state. There is the short-range repulsive interaction represented by $C_c=-1/6$ in Eq. (63). If the linear interaction between the Q and $\bar{Q}$ is strong enough, a bound color-octet state may be present in the pocket in the intermediate distance range to feed to the observed color-singlet state. The bound color-octet state will most likely be an $L'=0$ state rather than $L'=1$ state because the latter lies at a much higher energy. A bound color-octet $L'=0$ state will make the transition to the color-singlet state by an $M1$ transition, and there will be no $E1$ transition in this case.

Due to a lack of knowledge for bound color-octet states at present, we shall consider color-octet states to exist only in the continuum and shall assume only the color-Yukawa interaction with no linear interaction. Using such a color-octet potential, we obtain the wave function in the continuum using methods outlined in Sec. III. The K factor can be evaluated to give the square of the L'th derivative of the wave function of the continuum color-octet state at the origin [Eq. (75)]. The wave functions in the initial color-octet state and the final color-singlet state can be used to evaluate the matrix elements of $M1$ and $E1$ operators. We then obtain the various quantities in Eq. (90). For the purposes of displaying the results, we introduce the product F_R defined as

$$F_R = \left| C_{J'L'S'} \sqrt{\frac{(2L'+1)!!}{4\pi L'!}} \frac{1}{M_Q^{L'}} \left[\frac{d^{L'} R_{J'L'S'}(r)}{dr^{L'}} \right]_{r\to 0} \right|^2 . \quad (91)$$

There is a limit for the maximum energy of an emitted soft gluon. One envisages that gluons with an energy higher than ω_g(cutoff) will be emitted at a shorter time scale and will be described by perturbative QCD and not by the soft gluon emission through an intermediary nonperturbative
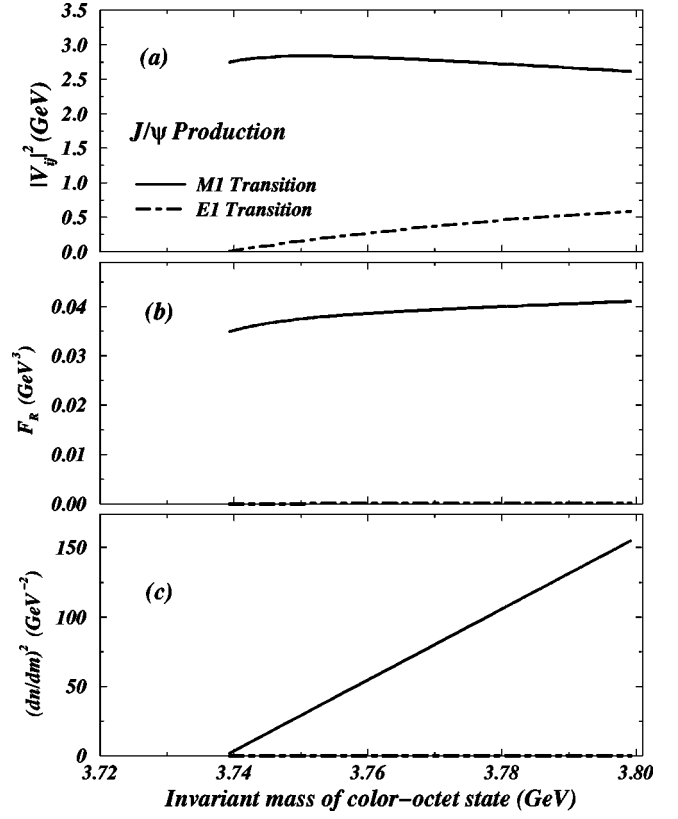

FIG. 3. $|V_{ij}|^2$, F_R, and $(dn/dm)^2$ as a function of the invariant mass $m_{J'L'S'}$ of the intermediate color-octet state for the production of J/ψ.

state. The cutoff of this soft gluon energy depends on the size of the bound state, as the time required for a gluon to travel the length of the radius $R/2$ of a J/ψ corresponds to an energy of $2\hbar/R \sim 0.8$ GeV. (Here, R is the separation between Q and $\bar{Q}$ which is about 0.5 fm for J/ψ.) Therefore the maximum energy of an emitted soft gluon should be of the order of 0.8 GeV. The extracted color-octet matrix element depends on this cutoff gluon energy. We find that a value of $\omega_g(\text{max})=0.7$ GeV gives the best description and is approximately consistent with this estimate of $2\hbar/R$ from the size consideration. For this maximum value of soft-gluon energy, the invariant mass of the color-octet state is 3.8 GeV.

In Fig. 3 we show $|V_{ij}|^2$, F_R, and $(dn/dm)^2$ as a function of the invariant mass $m_{J'L'S'}$ of the intermediate color-octet state for the production of the J/ψ state. Corresponding quantities for the production of ψ' are shown in Fig. 4.

For the production of J/ψ with an $M1$ transition, the initial color-octet state is an $L'=0$ state. The results of Fig. 3 indicate that the square of the matrix element $|V_{ij}|^2$ and F_R are approximately constants as a function of the color-octet invariant mass. The density of states $dn/dm_{J'L'S'}$ for $L'=0$ increases with the invariant mass of the color-octet state. For the $E1$ transition leading to the production of J/ψ, color-octet states making the $E1$ transition are $L'=1$ states. The quantity $|V_{ij}|^2$ increases monotonically from zero as the invariant mass increases. The factor F_R for the $E1$ transition is small because it is proportional to v^2 [Fig. 3(b)]. Because of the centrifugal barrier, the density of states $dn/dm_{J'L'S'}$ for
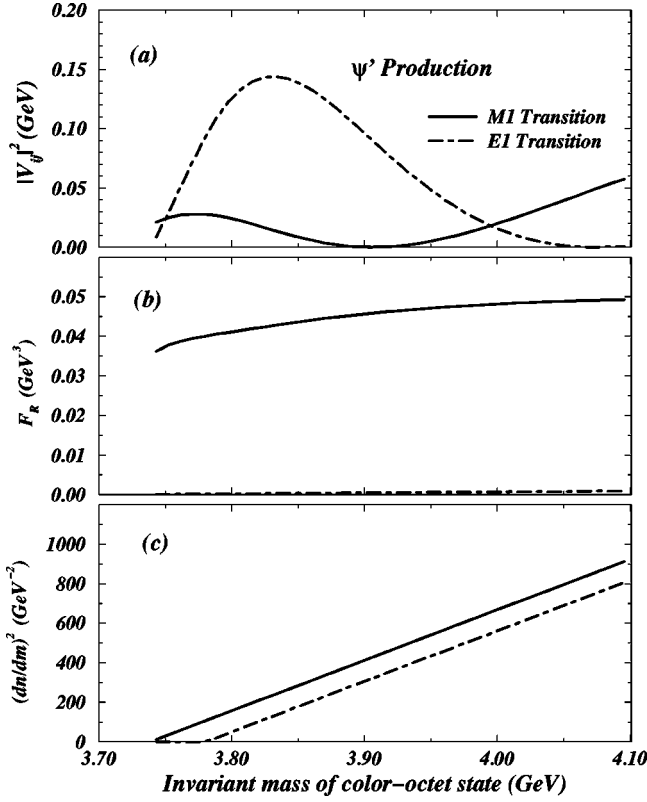
FIG. 4. $|V_{ij}|^2$ and F_R as a function of the invariant mass $m_{J'L'S'}$ of the intermediate color-octet state for the production of the ψ' state.

$L'=1$ is zero for color-octet states in the range of invariant mass up to 3.8 GeV under consideration [Fig. 3(c)]. A higher energy, $L'(L'+1)/M_Q R^2$, is needed for the Q and $\bar{Q}$ to overcome the centrifugal barrier to come to a distance of R ~ 0.5 fm for these angular momentum $L'=1$ states. As a consequence, there is no $E1$ transition from the color-octet $L'=1$ state to the J/ψ state in the region of soft gluon emission. The $M1$ transition dominates over the $E1$ transition for J/ψ production. The color-octet matrix elements for various transitions are given in Table II. We obtain $\langle \mathcal{O}_8^{J/\psi}(^3S_1^{(8)})\rangle$ $=0.076$ GeV^3 and 0 for $\langle \mathcal{O}_8^{J/\psi}(^3S_1^{(8)})\rangle$, with the combination $\langle \mathcal{O}_8^{J/\psi}(^3S_1^{(8)})\rangle + (3/M_Q^2)\langle \mathcal{O}_8^{J/\psi}(^3S_1^{(8)})\rangle = 0.076$ GeV^3, which can be compared with the value of 0.066 extracted from the CDF measurements.

For the production of ψ', the size estimate gives a maximum gluon energy of the order of $2\hbar/(1\text{ fm}) \sim 0.4$ GeV which corresponds to a maximum invariant mass of about 4.1 GeV for the color-octet state. For the production of ψ', the radial wave function of the final state has a node and a change of the sign of the wave function. As a result, the matrix element V_{ij} for the $M1$ transition involves a high degree of cancellation and the magnitude of the matrix element is small for the $M1$ transition [Fig. 4(a)]. For the $M1$ transition the F_R factor is approximately a constant but $(dn/dm_{J'L'S'})^2$ increases monotonically with $m_{J'L'S'}$ (Fig. 4). For the $E1$ transition $|V_{ij}|^2$ depends on the invariant mass of the color-octet state and is much larger than that for the $M1$ transition at $m_{J'L'S'}=3.85$ GeV. On the other hand, the F_R factor is proportional to k^2/M_Q^2 [Eq. (75)] for the $E1$ transition and is small in magnitude [Fig. 4(b)]. The square of the density of the $L'=1$ states, $(dn/dm)^2$, increases with the color-octet invariant mass after a threshold. The combined result is a small $E1$ transition probability for the production of ψ' as shown in Table II. We obtain $\langle \mathcal{O}_8^{\psi'}(^1S_0)\rangle$ $=0.015$ GeV^3 for the $M1$ transition, and $\langle \mathcal{O}_8^{\psi'}(^3P_0)\rangle/M_Q^2$ $=0.0001$ GeV^3 for the $E1$ transition. The $M1$ transition probability is again much greater than the $E1$ transition probability. The sum $\langle \mathcal{O}_8^{\psi'}(^1S_0)\rangle + 3\langle \mathcal{O}_8^{\psi'}(^3P_0)\rangle/M_Q^2$ agrees approximately with the matrix element extracted in CDF measurements [5].

It is easy to show that the probability for the transition $^1S_0^{(8)} \rightarrow (^3S_1^{(1)}, S_z)$ is the same for $S_z=-1$, 0, and 1. Therefore, an initial $^1S_0^{(8)}$ state will lead to an equal population of the final magnetic substates of $^3S_1^{(1)}$ quarkonium and consequently gives an isotropic angular distribution of muons from the decay of the quarkonium.

In J/ψ and ψ' production at fixed-target energies, the fusion of two gluons is the dominant mechanism of direct J/ψ and ψ' production and gives rise to the color-octet $^1S_0^{(8)}$ and $^3P_J^{(8)}$ states. Thus, the direct production of J/ψ and ψ' bound states by the emission of very soft gluons occurs mainly through the $M1$ $^1S_0^{(8)} \rightarrow {}^3S_1^{(8)}$ transition, and the produced $^3S_1$ bound state will be nearly unpolarized, with an approximately isotropic angular distribution for the decay muons. Assuming $\langle \mathcal{O}_8^{J/\psi}(^3P_J)\rangle/M_Q^2=0$ and including the small contributions from the color-singlet mechanism and $q\bar{q}$ annihilation, Beneke et al. [6] obtain $\lambda=0.15$ for the muon angular distribution $1+\lambda\cos^2\theta$ for ψ' decay in the

TABLE II. Color-octet matrix elements from Eq. (58).

Matrix element	Matrix elements from CDF measurements	Matrix elements calculated with Eq. (58)
$\langle \mathcal{O}_8^{J/\psi}(^1S_0)\rangle$		0.076 GeV^3
$\langle \mathcal{O}_8^{J/\psi}(^3P_0)\rangle/M_Q^2$		0
$\langle \mathcal{O}_8^{J/\psi}(^1S_0)\rangle + 3\langle \mathcal{O}_8^{J/\psi}(^3P_0)\rangle/M_Q^2$	0.066 GeV^3	0.076 GeV^3
$\langle \mathcal{O}_8^{\psi'}(^1S_0)\rangle$		0.015 GeV^3
$\langle \mathcal{O}_8^{\psi'}(^3P_0)\rangle/M_Q^2$		0.0001 GeV^3
$\langle \mathcal{O}_8^{\psi'}(^1S_0)\rangle + 3\langle \mathcal{O}_8^{\psi'}(^3P_0)\rangle/M_Q^2$	0.018 GeV^3	0.015 GeV^3

Gottfried-Jackson frame, which falls within the error of the experimental value of $\lambda = 0.02 \pm 0.14$ for the ψ' at $\sqrt{s} = 21.8$ GeV [14]. The experimental λ value for direct J/ψ production is not yet available. The measured λ value for total J/ψ production is 0.028 ± 0.004 at $\sqrt{s} = 15.3$ GeV [16] and is -0.02 ± 0.06 for $\sqrt{s} = 21.8$ GeV [15], which includes direct and indirect J/ψ production from the decay of χ states. The theoretical calculation of λ for the total J/ψ yield is still incomplete because of the difficulty of the color-octet model to produce the observed χ_1/χ_2 ratio [6].

We now return to the question of the universality of the matrix elements. With $\langle \mathcal{O}_8^{3S_1}(^1S_0) \rangle \gg \langle \mathcal{O}_8^{3S_1}(^3P_J) \rangle / m_c^2$, the discrepancy between the matrix elements in the CDF measurement and the fixed-target measurements is reduced to a factor of 2 for J/ψ and 4 for ψ' [6]. The discrepancy can be further reduced when one takes into account the physical masses and allows for the finite energy carried by the soft gluon, as suggested by Beneke *et al.* [6].

VII. CONCLUSIONS AND DISCUSSIONS

For the color-octet model to be a valid description, we need to understand the soft gluon emission process. As the soft gluon emission occurs at a late stage of the production process, the final state can be described in terms of an interacting Q-$\bar{Q}$ potential. We use a potential model to describe the initial and final states in the soft gluon emission process.

In our discussions of the long-distance behavior of the Q-$\bar{Q}$ potential, it is necessary to take into account the effects of the spontaneous production of light quarks as the heavy Q and $\bar{Q}$ pull apart. Accordingly, the potential at large distances is screened due to the breaking of the string. A potential which has such a property was used successfully to describe the color-singlet charmonium bound states and resonances.

Not much is known about the potential between Q and $\bar{Q}$ in a color-octet state except that it is repulsive at short distances. The combined potential may have a pocket deep enough to hold a bound state. A bound color-octet state is a relatively long-lived object having interesting experimental signatures such as the occurrence of chains of pions in the rapidity gap between the produced quarkonium state and its complementary partner as the octet is pulling apart from its complementary color partner.

Due to a lack of knowledge of color-octet bound states at present, we consider color-octet states in the continuum. For our studies, we assume a weak color-octet interaction which contains only the repulsive color-Yukawa interaction at short

distances to give illustrative results concerning the production process.

We found that the production cross section depends on three factors: the magnitude of the $M1$ and $E1$ matrix elements, the wave function or its derivatives at the origin, and the density of the color-octet states in the soft-gluon emitting region. It turns out that for J/ψ production, the magnitude of the $M1$ matrix elements is relatively constant for the range of soft-gluon emissions, while the magnitude of $E1$ transition matrix elements increases monotonically from zero as the energy of the soft gluon increases. The $E1$ transition is, however, highly inhibited because within the range of low energy soft-gluons (up to $\omega_g \sim 0.8$ GeV), the density of the initial color-octet $L' = 1$ states is zero as a higher energy is needed to overcome the centrifugal barrier. As a consequence, the $M1$ transition dominates over the $E1$ transition.

For ψ' production, the transition matrix element for the $M1$ transition is small because of the cancellation involving a final state whose wave function changes sign in different regions of r. However, k^2/M_Q^2 is small and limits the contribution of the $E1$ transition. The $M1$ transition is again the dominant mode of transition.

It is clear from the above discussions that the dominance of the $M1$ over the $E1$ transition arises mainly from the limiting factor of the density of P-wave color-octet states and k^2/M_Q^2. Thus, the dominance of $M1$ soft gluon emission over the $E1$ transition is a rather general property which is not affected by a moderate variation of the strength of the Q-$\bar{Q}$ potential. On the other hand, the magnitude of the $M1$ cross section is sensitive to the maximum soft gluon energy and the Q-$\bar{Q}$ interaction.

In conclusion, while there are questions concerning the color-octet model, some of these may be resolved by careful refinements of the details of the model. In particular, the dominance of the $M1$ transition $^1S_0^{(8)} \to {}^3S_1^{(1)}$ over the $E1$ transition $^3P_J^{(8)} \to {}^3S_1^{(1)}$ can explain the isotropic muon angular distribution and reduce the discrepancies of the matrix elements between the CDF and the fixed-target measurements. There may be the universality of the color-octet matrix elements when one takes into account the physical masses and the finite energies of the soft gluon.

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